

Nambu Jona-Lasinio Model of $\bar{q}q$ Bose Einstein Condensation and pseudogap phase

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We show the existence of a pseudogap phase in the Nambu Jona-Lasinio model of quark interactions. In the pseudogap phase chiral symmetry is restored but $q\bar{q}$ pseudoscalar mesons still exist and they are massive. Such a behavior is intermediate between a BCS superconductor and a Bose Einstein Condensate. We suggest the relevance of this phenomenon for an understanding of recent lattice QCD and experimental data.

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I. INTRODUCTION

Our current understanding of nuclear phenomena indicates that, at finite temperature, hadronic matter undergoes a phase transition to deconfined gluons and quarks. Quantum-Chromo-Dynamics (QCD) lattice calculations [1] strongly support the conclusion that, at some critical temperature, a transition to the quark-gluon plasma (QGP) phase occurs; this temperature is numerically equal to the critical temperature for the restoration of the chiral symmetry.

The earliest suggested signature of QGP was the strong suppression of the light and heavy $q\bar{q}$ bound states for temperatures larger than critical temperature [2]. However, more recent lattice results [3], using the Maximal Entropy Method, have found that mesonic bound states, light or heavy, actually persist for temperature at least a factor of two larger than the critical temperature and the quasiparticle mass turns out to be still quite large [4].

Furthermore, the RHIC data on the radial and elliptic flows [5] can be explained by partonic cascades [6] and viscosity corrections [7] only if the partonic cross section is about 50 times larger than the perturbative QCD calculations, which indicates a strong coupling regime.

Some models have been proposed to describe this strongly interacting phase [8, 9]. The general feature resulting from both theory and experiment is the appearance of two different temperatures: T_c and T^* ($T_c < T^*$). The lower temperature T_c should be associated with deconfinement and chiral symmetry restoration, the upper T^* is related to the decoupling of the bound states from the spectrum [3].

The presence of two temperature scales is an interesting phenomenon that has an analogue in high temperature superconductors. The nature of the phase transition for these systems is still matter of debate, see, for a recent review [10]. An established experimental fact seems however to be the existence in these superconductors of a pseudogap [11], which is a depletion of the single particle density of states around the Fermi level. Another characteristic feature of the high T_c superconductors, is their coherence length ξ_0 which is much smaller than in ordinary superconductors. Both features might be the phenomenological manifestations of a crossover from the Bardeen-Cooper-Schrieffer (BCS) behavior of the ordinary superconductors to a Bose Einstein Condensate (BEC) behavior [12]. In this scheme at a certain critical temperature a pseudogap phase is reached where the quasiparticles are still gapped although phase coherence and long range order are lost. Superconducting fluctuations might be responsible for this phenomenon, though other interpretations are also possible [10]. The relation between the pseudogap transition and field fluctuations has been qualitatively elucidated in the Nambu Jona-Lasinio (NJL) model [13, 14]; moreover it has been discussed in [15] in the framework of a semi-phenomenological constituent quark model, in [16] in the case of color superconductivity and in [17] for low density nuclear matter.

In this paper we analyze in more detail the dynamics of this NJL pseudogap phase, described by fermionic degrees of freedom and/or by the equivalent bosonic system, including also finite density effects. The picture that emerges from an explicit numerical calculation can be summarized as follows:

- i) Chiral symmetry is broken in the chiral limit, at $T < T_c$ and small density. The scalar meson, σ , is massive and the pseudoscalar meson, π , is the massless Nambu Goldstone Boson (NGB).
- ii) The temperature (and/or the density) restores the chiral symmetry and the order

parameter $\langle \bar{\psi}\psi \rangle$ goes to zero at T_c . The signal of the restoration of the chiral symmetry, due to the background fluctuations, is given by the equal mass of the scalar and pseudoscalar mesons for $T > T_c$. Therefore the NGB acquires a mass, while remaining in the spectrum. In this phase the constituent fermion mass remains finite (pseudogap regime) and there is a strong fermion-meson coupling.

iii) Above another critical temperature, $T^* > T_c$, the constituent fermion mass goes to zero and the mesons decouple from the physical spectrum.

The paper is organized as follows. In section II we present our formalism. In section III we discuss the bosonization of the NJL model and in section IV we compute the stiffness. In section V we compute the pseudogap and in section VI we draw our conclusions.

II. CONSTITUENT QUARK MASS AND PION DECAY CONSTANT

We use

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_4 \quad (1)$$

with massless quarks at finite chemical potential in two flavors ($N_f = 2$). Here

$$\mathcal{L}_0 = \bar{\psi}(i\partial_\nu\gamma^\nu + \mu\gamma_0)\psi \quad (2)$$

while \mathcal{L}_4 gives the Nambu Jona-Lasinio (NJL) coupling ([18] and for reviews [19], [20]):

$$\mathcal{L}_4 = \frac{G_0}{2N_c} \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2 \right] . \quad (3)$$

In the sequel we neglect the so called exchange terms [19]. In the Mean Field Approximation (MFA) the constituent quark mass is obtained by the gap equation, which at $T = \mu = 0$ is as follows

$$m_q = 4N_fN_c \frac{G_0}{2N_c} \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{m_q}{\sqrt{p^2 + m_q^2}} , \quad (4)$$

where Λ is a 3D cutoff. One can note the relation between the constituent quark mass and the chiral condensate

$$m_q = - \frac{G_0N_f}{N_c} \langle 0 | \bar{u}u | 0 \rangle . \quad (5)$$

Going beyond the MFA means considering diagrams of order $\left(\frac{G_0}{2N_c}\right)^2$, see e.g. [21]. Clearly the smaller N_c , the larger the role of the fluctuations. As a consequence, an effective momentum dependent quark mass is introduced $M_{eff}(p)$. The momentum dependence varies

with the model, but for the model we use here, which corresponds to model *A* of [21], $M_{eff}(p)$ is almost constant, with variations of a few percent in the whole p range. Therefore we will neglect these effects altogether. Moreover it can be observed that the reliability of the mean field gap equation to evaluate the constituent quark mass also in a region where the fluctuations are relevant (as for example for small N_c), has been explained by Witten [22] in exactly solvable models in 1+1 dimension and numerically checked by lattice calculations in 2+1 dimensional models [23].

The gap equation at finite T and μ can be obtained by considering a sum over Matsubara frequencies, by the substitution of the energy integration variable p^0 with $i\omega_n - \mu$, where μ is the quark chemical potential and $\omega_n = \pi T(2n + 1)$.

Eq. (4) relates the constituent quark mass m_q at $T = \mu = 0$ to the NJL coupling G_0 and the cutoff Λ . Its generalization for finite T and μ provides the T and μ -dependent constituent quark mass in the mean field approximation: $m_q(T, \mu)$. By getting rid of the NJL coupling constant, we get $m_q(T, \mu)$ from the constituent mass at $T = \mu = 0$ as follows:

$$0 = \int_{\Lambda} \frac{d^3p}{(2\pi)^3} \left[\frac{1}{\sqrt{p^2 + m_q^2}} - \frac{\sinh y}{\epsilon (\cosh y + \cosh x)} \right] \quad (6)$$

where

$$x = \frac{\mu}{T}, \quad y = \frac{\epsilon}{T}, \quad \epsilon = \frac{\sqrt{p^2 + m_q^2(T, \mu)}}{T}. \quad (7)$$

The pion decay constant at $T = \mu = 0$ is

$$f_{\pi}^2 = -4im_q^2 N_c \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_q^2 + i\epsilon)^2}. \quad (8)$$

One can use this equation to get Λ by fixing the constituent quark mass. Using $m_q = 300$ MeV as an input, from $f_{\pi} = 93.3$ MeV one gets $\Lambda = 675$ MeV.

We define the $T^* = T^*(\mu)$ temperature by $m_q(T^*, \mu) = 0$; numerically one finds, at $\mu = 0$, $T^* = 185$ MeV. For generic values of μ the result can be obtained by Eq. (6). The generalization of Eq. (8) to finite T and μ is straightforward.

III. BOSONIZATION OF THE NJL

Let us first work at $T = \mu = 0$. As is well known the NJL model can be made equivalent to the linear σ model [24] (see also [19]). One introduces fields σ and $\boldsymbol{\pi}$ by writing the

generating functional as follows

$$Z = \int [d\psi][d\bar{\psi}] \exp \left\{ \int dx (\mathcal{L}_0 + \mathcal{L}_4) \right\} = \int [d\psi][d\bar{\psi}][d\sigma][d\pi^i] \\ \times \exp \left\{ \int dx \left(\bar{\psi} (i\partial_\nu \gamma^\nu + \mu\gamma_0 - g_0(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})) \psi - \frac{g_0^2 N_c}{2G_0} [\sigma^2 + \boldsymbol{\pi}^2] \right) \right\} \quad (9)$$

where

$$g_0 = \frac{m_q}{f_\pi} \quad (10)$$

is the meson-quark coupling constant. Eq. (10) is the analogous of the Goldberger-Treiman relation. By bosonization and derivative expansion the NJL model becomes equivalent to the σ model with lagrangian

$$\mathcal{L}_\sigma = \frac{\beta}{2} \left((\partial\sigma)^2 + (\partial\boldsymbol{\pi})^2 + \frac{\kappa^2}{4} (\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 \right). \quad (11)$$

The parameter β is given by [24], [13]:

$$\beta = 4g_0^2 N_c N_f (I_0 - 2\Omega_0), \quad (12)$$

with

$$I_0 = \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{(p_E^2 + m_q^2)^2}, \\ \Omega_0 = \frac{1}{4} \int \frac{d^4 p_E}{(2\pi)^4} \frac{p_E^2}{(p_E^2 + m_q^2)^3}. \quad (13)$$

Classically one has $\langle \pi_a \rangle = 0$, $\langle \sigma \rangle = f_\pi$; as a consequence the field $\sigma' = \sigma - f_\pi$ acquires a mass $\kappa f_\pi / \sqrt{2}$ while the pions remain massless.

If the σ' mass is very large, the model is equivalent to the non linear σ model with action

$$S = \frac{\beta}{2} \int d^4 x \left((\partial\sigma)^2 + (\partial\boldsymbol{\pi})^2 \right) \quad (14)$$

and fields satisfying the constraint

$$\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2. \quad (15)$$

IV. STIFFNESS

The generating functional of the nonlinear σ model for vanishing external sources can be written as follows

$$Z = \int [d\sigma][d\boldsymbol{\pi}][d\lambda] \exp\{iS\} \quad (16)$$

$$S = \frac{\beta}{2} \int d^D x \left((\partial\sigma)^2 + (\partial\boldsymbol{\pi})^2 - \lambda (\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2) \right) \quad (17)$$

where the λ functional integration implements the non linear condition on the fields (15).

After integration over the $\boldsymbol{\pi}$ fields we get

$$Z = \int [d\sigma][d\lambda] \exp(iS_{eff}[\sigma, \lambda]) \quad (18)$$

with

$$S_{eff}[\sigma, \lambda] = \frac{i}{2} \text{Tr} \ln[\partial^2 + \lambda(x)] + \frac{\beta}{2} \int d^D x \left(f_\pi^2 \lambda(x) - \sigma[\partial^2 + \lambda(x)]\sigma \right) . \quad (19)$$

If the number of flavors is large we can use the saddle point approximation and search for solutions σ, λ independent of x . Since we are looking for phase transitions at finite T and μ , we consider Matsubara frequencies and non vanishing baryonic chemical potential. One gets in this way [25]:

$$\begin{aligned} 0 &= \beta(f_\pi^2 - \sigma^2) - (N_f^2 - 1) \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \frac{T}{(\tilde{\omega}_n - i\mu)^2 + k^2 + \lambda} \\ 0 &= \lambda\sigma \end{aligned} \quad (20)$$

with $\tilde{\omega}_n = 2\pi nT$; therefore one has two solutions [25]:

- i) $\sigma = 0$ and λ implicitly given by

$$\beta f_\pi^2 = (N_f^2 - 1) \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \frac{T}{(\tilde{\omega}_n - i\mu)^2 + k^2 + \lambda} \quad (21)$$

- ii) $\lambda = 0$ and

$$\sigma^2 = f_\pi^2 - \frac{N_f^2 - 1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \frac{T}{(\tilde{\omega}_n - i\mu)^2 + k^2} \quad (22)$$

The case i) corresponds to chiral symmetry restoration, the case ii) to spontaneous symmetry breaking and massless pions. Note that in the chiral symmetric phase, $\sigma = 0$ and also $\langle 0|\bar{q}q|0\rangle = 0$.

Let us now consider the case of finite T and μ . The critical line of chiral symmetry restoration is defined by $\lambda = \sigma = 0$, or

$$\begin{aligned} \beta_c &= \frac{N_f^2 - 1}{f_\pi^2(T, \mu)} \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \frac{T}{(\tilde{\omega}_n - i\mu)^2 + k^2} \\ &= \frac{N_f^2 - 1}{f_\pi^2(T, \mu)} \frac{1}{8\pi^2} \int_0^{\Lambda_\pi} dk k \left[\coth\left(\frac{\mu + k}{2T}\right) - \coth\left(\frac{\mu - k}{2T}\right) \right] \end{aligned} \quad (23)$$

At finite temperature and density we have for the stiffness, from Eqns. (12), (13)

$$\beta = 2g_0^2 N_f N_c T \sum_{n=-\infty}^{+\infty} \int_{\Lambda} \frac{d^3 p}{(2\pi)^3} \left[\frac{1}{[\epsilon^2 + (\omega_n - i\mu)^2]^2} + \frac{m_q^2(T, \mu)}{[\epsilon^2 + (\omega_n - i\mu)^2]^3} \right]. \quad (24)$$

The critical temperature T_c as a function of μ is obtained by equating (23) and (24) [13], [14]. The value of T_c depends on the value of the pionic cutoff Λ_π ; typical values should be of a few units of f_π since f_π fixes the scale of the derivative expansion. Numerically we find, at $\mu = 0$, for $\Lambda_\pi = 200$ MeV, $T_c = 146$ MeV; for $\Lambda_\pi = 300$ MeV, $T_c = 127$ MeV. We stress the assumption of two different cutoffs, Λ and Λ_π ; for a discussion on this point see VI.

If $T_c(\mu)$ defines the critical line, we can compute the mass of the composite boson formed by the $q\bar{q}$ pair above $T_c(\mu)$, i.e. in the Wigner phase. The existence of such pairs in the NJL model is a feature that was observed long ago [26] and interpreted as precursor of chiral phase transition at finite temperature and chemical potential. We differ from these authors for two important aspects. First, we identify the critical temperature as T_c and not as T^* . Second, in [26] the fluctuations are identified by looking at peaks in the ω distribution of the strength function

$$\mathcal{A}(\mathbf{k}, \omega) \propto \text{Im} G^R(\mathbf{k}, \omega) \quad (25)$$

at $\mathbf{k} = 0$ (G^R is the retarded Green function describing the fluctuation of the order parameter in the Wigner phase). Here we use the stiffness to take into account the fluctuations and compute the mass of the π mode $m_\pi^2 = \lambda$ by identifying $\beta(\lambda)$ in (21) with β given by (24).

For $\mu = 0$ the results are reported in fig. 1 for the value of the parameter $\Lambda_\pi = 200$ MeV and for $N_c = 3$ (solid lines) and $N_c = 10$ (dashed lines). Before discussing the dependence on N_c , let us note that in fig. 1 the function $\sigma(T)$ differs from zero in the interval $(0, T_c)$ and vanishes in (T_c, T^*) for both values of N_c . On the contrary the function $m_\pi(T)$ vanishes in $(0, T_c)$ and rapidly increases in (T_c, T^*) with a divergent behavior around T^* .

An interesting aspect of these results is that they allow to define a cross-over temperature $T_{pair} > T_c$; in our computation T_{pair} is the temperature at which the composite bosons disappear from the physical spectrum. As a result of our approximation we find $T_{pair} \simeq T^*$. Lattice QCD investigations show that the deconfinement and chiral phase transitions take place at the same temperature, see e.g. [27] and [28]. Though our results indicate the existence of two different temperatures, they do not contradict lattice studies. As a matter of fact, the NJL model does not incorporate confinement. Therefore we can only state that our lower critical temperature T_c is the chiral restoration critical temperature, while leaving

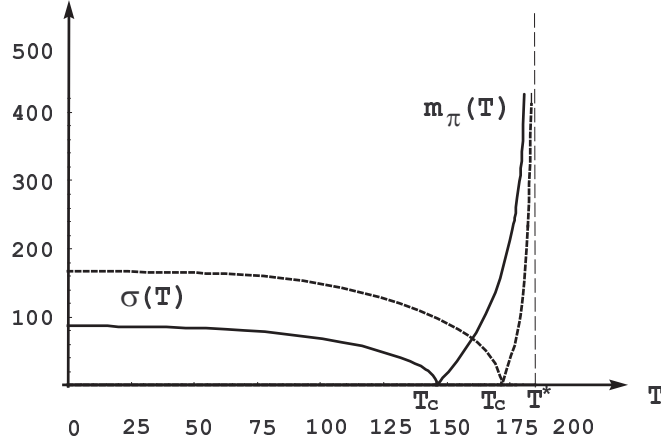


FIG. 1: - The parameter σ and mass of the pionic mode m_π as functions of the temperature T at $\mu = 0$ (in MeV). T_c is the critical temperature. For $T > T_c$ σ vanishes, while m_π vanishes for $T < T_c$. Values of the parameters are $\Lambda_\pi = 200$ MeV, $\Lambda = 675$ MeV. Solid lines refer to the case $N_c = 3$, dashed lines to $N_c = 10$ (weaker coupling). As a result of the computation, $T_c \approx 146$ MeV for $N_c = 3$ and $T_c \approx 172$ MeV for $N_c = 10$. $T^* \simeq T_{pair}$ is the dissociation temperature of the $\bar{q}q$ pairs.

undecided if it coincides with the deconfinement temperature, defined for example as the temperature where the string tension vanishes. As for the upper characteristic temperature $T_{pair} \simeq T^*$, it is unrelated with the deconfinement temperature and is defined either as the temperature where the strongly correlated Cooper pairs, behaving as massive mesonic states in the interval (T_c, T^*) , decouple from the spectrum, or as the temperature where the pseudogap phase disappears.

We conclude that the behavior of this model is more akin to a Bose-Einstein condensate than to a BCS system. Indeed while in the latter case the temperature characterizing pair formation coincides with the condensation temperature, the former case is generally characterized by two critical temperatures, one (upper) temperature when the pairs are preformed, and another (lower) where the pairs condense (for discussions in condensed matter see e.g. [29] and for a review [10]). This behavior is confirmed by the large N_c limit of the present model. In this limit the four fermion coupling becomes weak, the mean field approximation is exact and the two temperatures merge. This behavior can be easily proved by taking

the large N_c limit of the equation for the stiffness that determines T_{pair} . For $N_c \rightarrow \infty$ one gets $T_c \simeq T_{pair}$, and the model defines a BCS superconductor. By way of example we have reported in Fig. 1 the result of the calculations for $N_c = 10$ (dashed lines) showing that for weak coupling $T_c \rightarrow T^*$.

V. PSEUDOGAP

The pseudogap can be seen by the density distribution $N(\omega)$:

$$N(\omega) = 4 \int \frac{d\mathbf{k}}{(2\pi)^3} \text{Tr}_{c,f} \rho^0(\mathbf{k}, \omega) \quad (26)$$

(c, f =color, flavor), where

$$\rho^0(\mathbf{k}, \omega) = \frac{1}{4} \text{Tr}_d \gamma^0 \mathcal{A}(\mathbf{k}, \omega) \quad (27)$$

(d =Dirac) and

$$\mathcal{A}(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} G^R(\mathbf{k}, \omega) . \quad (28)$$

$G^R(\mathbf{k}, \omega)$ is the analytic continuation of the imaginary time Green function \mathcal{G} , in other words

$$G^R(\mathbf{k}, \omega) = \left[G^{-1}(\mathbf{k}, \omega + i\epsilon) - \Sigma_R(\mathbf{k}, \omega) \right]^{-1} \quad (29)$$

where $G^{-1}(\mathbf{k}, \omega) = (\omega + \mu)\gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} - m_q$ is the analytic continuation of the imaginary time Green function for free quarks $\mathcal{G}^{-1}(\mathbf{k}, \omega) = (i\omega_n + \mu)\gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} - m_q$. Notice that including only the mass term, i.e. at the lowest order in the interactions, we have

$$N(\omega) = N_0(\omega) = \frac{N_c N_f}{\pi^2} (\omega + \mu) \sqrt{(\omega + \mu)^2 - m_q^2} . \quad (30)$$

We include the quark mass term in $G(\mathbf{k}, \omega)$ in the spirit of an effective lagrangian approach, where the four-fermion coupling modifies the free quark dispersion law and the interactions are provided by the fluctuating pion field. Actually the absence of interactions would imply also the vanishing of m_q and one would get, in this case,

$$N(\omega) = N_{free}(\omega) = \frac{N_c N_f}{\pi^2} (\omega + \mu)^2 . \quad (31)$$

To compute the effect of the interactions at the next order we consider the effective lagrangian displayed in eq. (9):

$$\mathcal{L}_{eff} = \bar{\psi} [i\partial_\nu \gamma^\nu + \mu\gamma_0 - g_0(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi - \frac{g_0^2 N_c}{2G_0} [\sigma^2 + \boldsymbol{\pi}^2] \quad (32)$$

together with the condition (15). At the lowest order in the pion field we have the following interaction lagrangian

$$\mathcal{L}_{int} = \bar{\psi} \left(\frac{g_0}{2f_\pi} \boldsymbol{\pi}^2 - i g_0 \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \right) \psi \quad (33)$$

together with the mass term $-g_0 f_\pi \bar{\psi} \psi = -m_q \bar{\psi} \psi$. At the lowest order in g_0^2 we find

$$\Sigma_R(\mathbf{k}, \omega) = i ((\alpha_1 + i\alpha_2)\gamma_0 + (\lambda_1 + i\lambda_2)\mathbf{k} \cdot \boldsymbol{\gamma} + \zeta_1 + i\zeta_2) \quad (34)$$

and $(\alpha_i, \lambda_i, \zeta_i)$ real):

$$\begin{aligned} \alpha_1 + i\alpha_2 &= -(N_f^2 - 1) T \sum_{n=-\infty}^{+\infty} \int_{\Lambda_\pi} [dq] g_0^2(\mathbf{q}^2) \frac{i\omega_n - \omega - \mu}{D} \\ \lambda_1 + i\lambda_2 &= -(N_f^2 - 1) T \sum_{n=-\infty}^{+\infty} \int_{\Lambda_\pi} [dq] g_0^2(\mathbf{q}^2) \frac{1 - \mathbf{q} \cdot \mathbf{k}/\mathbf{k}^2}{D} \\ \zeta_1 + i\zeta_2 &= (N_f^2 - 1) T \sum_{n=-\infty}^{+\infty} \int_{\Lambda_\pi} [dq] g_0^2(\mathbf{q}^2) \left[\frac{1}{2m_q} - \frac{m_q}{D} \right]. \end{aligned} \quad (35)$$

The measure $[dq]$ is

$$[dq] = \frac{d^3q}{(2\pi)^3} \frac{1}{(2\pi nT)^2 + \mathbf{q}^2 + m_\pi^2} \quad (36)$$

and

$$D = [\omega_n + i(\omega + \mu)]^2 + (\mathbf{k} - \mathbf{q})^2 + m_q^2 \quad (37)$$

with $\omega_n = 2\pi T(n + 1/2)$. We have included a form factor in the definition of g_0^2 :

$$g_0^2(\mathbf{q}^2) = g_0^2 \left(\frac{m_P^2}{m_P^2 + (2\pi nT)^2 + \mathbf{q}^2} \right)^2. \quad (38)$$

It corrects the large q^2 behavior of the loop diagrams and plays a role analogous to the regulator used in [25]. We take $m_P = 100$ MeV, but the exact numerical value is of no interest here as we are only interested in the qualitative behavior of the density of states. We note that the effect of the form factor is to reduce the perturbative contribution to the density distribution $N(\omega)$. As a matter of fact this perturbative term decreases with decreasing m_P , while for $m_P \rightarrow \infty$ the effect of the form factor vanishes. Our numerical choice, $m_P = 100$ MeV, only indicates an order of magnitude. Much larger values are excluded since they would extend the validity of the approximation beyond the kinematical range dictated by the neglect of higher order terms in the derivative expansion of eq. (11). We also note that a further effect of the form factor is to make the series in eq. (35) rapidly convergent, which allows an inversion between analytic continuation and Matsubara summation, see e.g. ref. [16], and related work in refs. [30, 31].

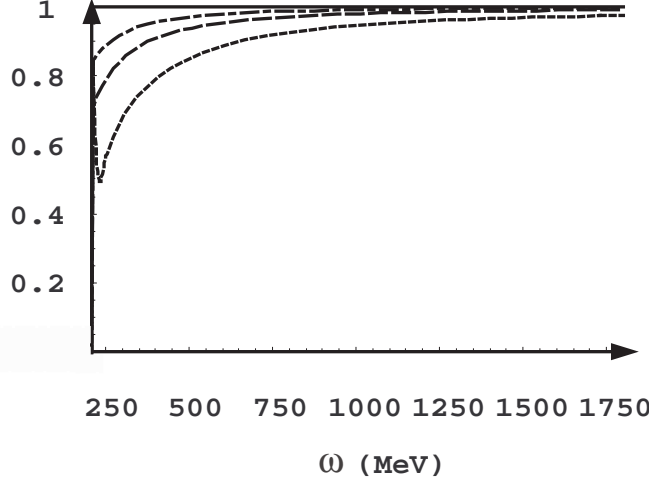


FIG. 2: - The ratio $N(\omega)/N_{free}(\omega)$ as a function of the energy ω . $N(\omega)$ is the density of states for the interacting theory; $N_{free}(\omega)$ the density for the free fields. Results are presented for $\mu = 10$ MeV and for various temperatures. The solid line represents the absence of interactions and corresponds to $T = T^*$. The other curves refer to three different values of $\delta = \frac{T - T_c}{T_c}$: $\delta = 0.02$, dotted; $\delta = 0.15$, dashed and $\delta = 0.20$, dotted-dashed line.

Similarly to the N_{free} case, the $|\mathbf{k}|$ integration can be performed using the residues theorem. Our results are as follows; we define $N(\omega) = N_0(\omega) + N_{pert}(\omega)$ and we get

$$N_{pert}(\omega) = \frac{N_c N_f}{\pi^2} \left(k_0 \alpha_2(\omega, k_0) + \frac{f(\omega, k_0)}{2k_0} + \frac{f'(\omega, k_0)}{2} \right). \quad (39)$$

Here f' denotes partial derivation in the k variable,

$$k_0 = \sqrt{(\omega + \mu)^2 - m_q^2} \quad (40)$$

and

$$f(\omega, k) = -2(\omega + \mu)[(\omega + \mu)\alpha_2(\omega, k) + k^2\lambda_2(\omega, k) + m_q\zeta_2(\omega, k)]. \quad (41)$$

Our results are displayed in Fig. 2 where we plot $\frac{N(\omega)}{N_{free}(\omega)}$ for three different values of the ratio

$$\delta = \frac{T - T_c}{T_c}, \quad (42)$$

i.e. $\delta = 0.02$ (dotted line) $\delta = 0.15$ (dashed line) and $\delta = 0.20$ (dotted-dashed line) and for one value of the chemical potential ($\mu = 10$ MeV). For other values of μ the results

are similar. The figure shows the existence of a pseudogap in the (T_c, T^*) region. The pseudogap is maximal near T_c and decreases for $T_c \rightarrow T^*$, which can be easily understood as the combined effect of the vanishing constituent mass m_q and the divergent behavior of m_π for $T \rightarrow T^*$.

VI. CONCLUSIONS AND OUTLOOK

The previous analysis shows that in the $T - \mu$ phase diagram of the NJL model, at a fixed value of μ (smaller than the critical value when the effects of color superconductivity become significant), there are two typical temperatures, associated with chiral symmetry restoration and with the dissociation of the $q\bar{q}$ pairs. This qualitative behavior is suggested in [14] by Babaev, who changes the conclusions of [13] employing a similar method. The authors of ref. [13] use $\Lambda_\pi \approx \Lambda$ and conclude that the NJL model has no chiral broken phase, unless the number of colors N_c is larger than its physical value $N_c = 3$. We agree with [14] where two different ultraviolet cutoffs are used, as they are related to different degrees of freedom. Our explicit calculations corroborate therefore the conclusions of [14]. In particular we explicitly prove the appearance of a pseudogap, related to the existence of a strong coupling. The existence of two different cutoffs, one fermionic (Λ) and another one bosonic (Λ_π) is due to the fact that, as shown in refs. [32] and [33], the regularization of quark loops by means of Λ does not restrict the four momenta of the mesons. In all non-renormalizable field theories, such as the NJL model, new parameters are introduced as more loops are included in the calculations. Since the inclusion of internal boson lines corresponds to add more loops, the presence of meson propagators implies the existence of the new cutoff Λ_π [32]. In [33] the interval $(0, 1.5)$ for Λ_π/Λ is investigated, but in [34] (see also [35]) it is shown that $\Lambda_\pi > \Lambda$ leads to instabilities. Our choice $\Lambda_\pi < \Lambda$ is consistent with [34]: in particular we find that the value $\Lambda_\pi = 200$ MeV produces a numerical value for the chiral phase transition in rough agreement with lattice results. We finally note that the existence of two different cutoffs can be also interpreted as the consequence of the non-universality of a critical stiffness in the non linear sigma model for 3+1 dimensions. This lack of universality implies [14] that the knowledge of β_c in the NJL model does not allow, by alone, to fix the position of the phase transition in the effective field theory. Therefore the appearance of a new parameter appears quite natural from this viewpoint. Summarizing: Our derivation is based, differently from

Ref. [13], on two cutoffs and, as such, it might be controversial. However, on the basis of the arguments of Refs. [32, 33, 34, 35] and [14] summarized above, we believe that using two different cutoffs is a proper procedure.

As already mentioned in the introduction, some other models have been proposed to understand in QCD the persistence of the $q\bar{q}$ bound states above the critical temperature and the hydrodynamical properties of the medium observed at RHIC; let us briefly comment on them. In [8] a strong QCD Coulomb-like interaction and the formation of a multiple bound states of quasiparticles has been suggested as a possible explanation of the data. However it is not clear if this strong Coulomb-like interaction is obtained in lattice simulations above the critical temperature [36]. More recently in [9] a strong non-confining $q\bar{q}$ potential is predicted in the framework of the field correlator method, which might account in part for the data.

The NJL model is obviously different from QCD; in particular it does not contain one of the key dynamical QCD effects, i.e. confinement. Nevertheless it contains the most relevant dynamics of the effective four fermion interactions and is largely used as an effective model of low energy QCD. Therefore the similarity between its behavior and some non-perturbative features of QCD (lattice results and the phenomenological interpretation of the RHIC data) should not cause much surprise. On the basis of the present work, it would be interesting to investigate if a pseudogap also shows up in lattice QCD at intermediate temperatures and small chemical potential.

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